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The use of different cost of equity models when valuing SMEs: A case study

El uso de diferentes modelos para determinar el coste de capital en la valoración de la PYME: Un caso de estudio

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Abstract
This study examines the valuation process of small and medium-sized enterprises (SMEs), using a Portuguese SME as a case study. The analysis focuses on calculating the cost of equity, with particular attention to the unique characteristics of these companies. The valuation was conducted using the discounted cash flow (DCF) method, with a preference for the average cost model. Two different approaches were employed to forecast free cash flows: (a) the geometric growth rate of sales, and (b) free cash flow projections derived from a model based on historical results. To calculate the cost of equity, three distinct models were used: (a) the capital asset pricing model (CAPM) with modifications proposed by Damodaran (2014), (b) the model developed by the Spanish Association of Accounting and Business Administration (AECA) from Rojo-Ramírez et al. (2012), and (c) the build-up model proposed by Ibbotson. These models serve as alternatives to the traditional CAPM, which is less suitable for unlisted companies due to the absence of a market beta. The study compares the results obtained from each model, focusing on their impact on the company's valuation. Valuing SMEs is crucial for enhancing corporate decision-making. Furthermore, the approaches utilized in this study provide valuable guidelines for financial analysts involved in SME valuation.

Keywords: company valuation; small and medium-sized enterprises; cost of capital; capital asset pricing model; CAPM; AECA model; Ibbotson model
JEL Classification: G32

Resumen
Este estudio examina el proceso de valoración de las pequeñas y medianas empresas (PYME), tomando como caso de estudio una PYME portuguesa. El análisis se centra en el cálculo del coste de los fondos propios, con especial atención a las características singulares de estas empresas La valoración se realizó utilizando el método del flujo de caja descontado (DCF), con preferencia por el modelo del coste medio. Para la previsión de los flujos de caja libres se emplearon dos enfoques diferentes: (a) la tasa de crecimiento geométrico de las ventas, y (b) las proyecciones de flujo de caja libre derivadas de un modelo basado en resultados históricos. Para calcular el coste de los fondos propios, se utilizaron tres modelos distintos: (a) el modelo de valoración de activos de capital (CAPM) con las modificaciones propuestas por Damodaran (2014), (b) el modelo desarrollado por la Asociación Española de Contabilidad y Administración de Empresas (AECA) a partir de Rojo-Ramírez et al. (2012), y (c) el modelo de acumulación propuesto por Ibbotson. Estos modelos sirven como alternativas al CAPM tradicional, que es menos adecuado para empresas no cotizadas debido a la ausencia de una beta de mercado. El estudio compara los resultados obtenidos en cada modelo, centrándose en su impacto sobre la valoración de la empresa. La valoración de las PYME es crucial para mejorar la toma de decisiones empresariales. Además, los enfoques utilizados en este estudio proporcionan valiosas directrices para los analistas financieros que intervienen en la valoración de las PYME.

Palabras clave: valoración de empresas; pequeñas y medianas empresas; coste del capital; modelo de valoración de activos de capital; CAPM; Modelo AECA; Modelo Ibbotson
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1. Introduction

Company valuations have become increasingly significant in the financial world, with businesses frequently seeking valuations of their own operations. These valuations enable companies to assess their value, providing a clearer understanding of their worth and facilitating future decision-making. Specifically, valuations play a critical role in decisions related to expansion, mergers, or even liquidation. Additionally, they help inform shareholders and investors about a company’s current value at a specific point in time, allowing comparisons with past valuations to determine whether the company’s value is increasing or decreasing.

The financial literature offers a wide range of techniques and models for valuation, which are generally designed for large, publicly traded companies. However, the valuation of privately held businesses, particularly small and medium-sized enterprises (SMEs), is equally important. SMEs are vital to the economies of countries worldwide. In the Portuguese context, for instance, SMEs represent the vast majority of companies (99.9%), according to Pordata statistics. This underscores the need to broaden the financial literature on the valuation of privately held companies, especially SMEs.

The aim of this study is to outline the valuation process of a privately held SME, using a Portuguese SME as a case study. This research seeks to address gaps in the literature dedicated to SME valuation by proposing alternative approaches that are more feasible and better suited to the unique characteristics of these businesses.

We have chosen to use the discounted cash flow (DCF) method, as it is widely recognized as the most accurate approach for estimating the intrinsic value of SMEs (Fernández, 2023). A critical variable in applying DCF models is the cost of capital, particularly the computation of the cost of equity. Typically, this is calculated using the capital asset pricing model (CAPM), which is the most widely used model in the financial world.

As a contribution to the existing literature, this study introduces three alternatives to the traditional CAPM to identify a more appropriate model for calculating the cost of equity in SME valuations. These alternatives include: (a) the CAPM model with modifications suggested by Damodaran (2014), (b) the three-component model proposed by the Spanish Association of Accounting and Business Administration (AECA), as outlined by Rojo-Ramírez et al. (2012), and (c) the build-up model proposed by Ibbotson, as discussed by Ballwieser and Wiese (2010).

These alternative models aim to address a key limitation of the CAPM model, particularly the cost of capital rate it generates. Despite its simplicity and widespread application, CAPM was not designed with private companies in mind, making it less suitable for the unique characteristics of SMEs. Thus, this study seeks to provide more viable and context-appropriate methods for SME valuation.

Following this introduction, Section 2 explores the theoretical and empirical framework of company valuation. This section is divided into eight subsections: (a) the differences between public and privately held companies, (b) the existing methods for valuing a company, (c) the discounted cash flow valuation method, (d) cash flow forecasts, (e) growth rates, (f) the existing models for calculating the cost of equity, (g) the cost of capital, and (h) the value of a company in perpetuity.

Section 3 presents the practical application of the case study to the company ABC, along with an analysis of the results obtained.

Finally, Section 4 concludes the paper by summarizing the findings from the case study, identifying the limitations encountered during the study, and offering suggestions for future improvements.

2. Theoretical framework

2.1 Valuation of listed and unlisted companies

According to Neves (2002), company valuation is the process of determining the value of a commercial, industrial, service, or investment unit for the purpose of conducting economic activities. Valuation can be performed for the company as a whole or for one of its business units, such as in cases of demerger, sale, or acquisition.

Vieito and Maquieira (2013) assert that the valuation of a company is based on determining its fair market value, which reflects its future potential. Similarly, Damodaran (2006) emphasized the importance of valuation, stating, “Valuation can be considered the heart of finance” (p. 694).

A key consideration in valuation is the distinction between publicly traded and privately held companies, as these differences significantly impact the valuation process. The primary difference lies in the tradability of shares: publicly traded companies have shares that are freely tradable, whereas privately held companies face restrictions on share tradability (Armour et al., 2009; de Jong, 2016).

The nature of the company—whether publicly traded or privately held—affects the selection of the appropriate valuation method (Adams & Thornton, 2009; Boudreaux et al., 2012). According to Abudy et al. (2016), privately held companies represent a substantial portion of the economy. However, estimating their cost of capital is significantly more complex compared to public companies (Boudreaux et al., 2012; Petersen et al., 2006).

One of the key distinctions between publicly traded and privately held companies lies in the availability of information. Information about privately held companies is far more limited compared to that of public companies. This difference arises because private companies are not required to disclose financial information publicly, whereas public companies are obligated to do so (Boudreaux et al., 2012; Capron & Shen, 2007; Damodaran, 2012; Petersen et al., 2006).

Another important consideration is the cost of capital, which tends to be higher for private companies (Abudy et al., 2016; Boudreaux et al., 2012). This is because investors in private companies typically lack the same level of diversification available to investors in public companies (Kartashova, 2014; Mueller, 2011). Consequently, private company investors are more exposed to risk (Boudreaux et al., 2012).

In light of Petersen et al. (2006) and Damodaran (2012), it is evident that applying “standard” techniques to estimate risk parameters, such as beta and standard deviation, is challenging for privately held companies. These techniques typically require market share prices, which are unavailable for private firms. The absence of such variables complicates the estimation of the cost of capital (Boudreaux et al., 2012).

Damodaran (2012) highlights that, while there are shared characteristics between publicly traded and privately held companies, significant differences must be accounted for when valuing them. Some of these differences include:

1. **Accounting standards and transparency**. Publicly traded companies are required to adhere to strict accounting standards, ensuring that their financial results are clearly and transparently presented in their financial statements. Conversely, privately held companies operate under more flexible standards, increasing the likelihood of manipulated results.
2. **Share prices and liquidity**. Publicly traded companies have associated share prices, which provide insights into their financial status. In contrast, private companies lack this information. Liquidity is another crucial factor; publicly traded companies benefit from higher liquidity, making it easier to liquidate positions. In privately held companies, liquidity is significantly lower, making liquidation more difficult and costly. Koeplin et al. (2000) and Boudreaux et al. (2012) argue that investors should receive an illiquidity discount to compensate for the higher risk associated with private companies.
3. **Mixing personal and business expenses**. Private companies often exhibit a blending of personal and business expenses. This occurs because the company owner is typically deeply involved in operations, with a substantial portion of their wealth invested in the business. This dynamic makes it difficult to separate personal and business interests.

Valuation plays a critical role in various areas of finance, including corporate finance, mergers and acquisitions, and portfolio management (Damodaran, 2012). Company valuation involves two main approaches: (a) the objective approach, where the company’s value is determined based on valuation models, and (b) the subjective approach, which relies on the appraiser’s experience and their ability to capture the nuances of reality (Fazzini, 2018).

2.2 Company valuation methodologies

The existing literature offers various approaches to determining which valuation models should be applied to companies, as outlined by Neves (2002), Damodaran (2012), and Fernández (2023). These approaches can be categorized into three main groups of valuation methodologies: (a) Discounted Cash Flow (DCF) Valuation, (b) Relative or Comparative Valuation (benchmarking against market values), and (c) Contingent Claim Valuation.

According to Aznar et al. (2016), the values derived from different valuation methods—when applied by various valuers or using widely accepted methodologies in the financial literature—tend to be very similar to each other. Furthermore, these values closely align with the figures obtained when the company is traded on the stock exchange.

2.2.1 Yield method

2.2.1.1 Discounted cash flow model

As highlighted by Neves (2002), Damodaran (2012), and Fernández (2023), the discounted cash flow (DCF) model enables the determination of a company's value by discounting future cash flows at a rate that reflects the associated risk. Copeland et al. (2000), Koeplin et al. (2000), Capinski and Patena (2009), Aznar et al. (2016), and Fernández (2023) all agree that this methodology is the most appropriate for valuing a company.

According to Damodaran (2012), the fundamental structure of the DCF model for private companies is not significantly different from that used for public companies. The key differences lie in how the model's input variables are calculated, particularly the discount rates and beta.

As discussed by Copeland et al. (2000), Neves (2002), and Fernández (2023), it is common practice to divide the valuation process into two distinct phases. The first phase, referred to as the forecast period, involves projecting the company's future cash flows, which may vary, along with the corresponding cost of capital. The second phase, known as the residual or terminal period, assumes that cash flows grow at a constant rate, as does the cost of capital (Heinrichs et al., 2013; Jennergren, 2008; Petersen et al., 2006).

The value calculated during the terminal period represents the net present value of all future cash flows accumulated beyond the end of the forecasting phase (Steiger, 2010). During this residual period, it is assumed that the company will continue to operate indefinitely (Fernández, 2023; Neves, 2002).

According to Copeland et al. (2000), there are five key steps to consider when valuing a company:

1. Analyze historical performance.
2. Project performance.
3. Estimate the cost of capital.
4. Estimate the value in perpetuity.
5. Calculate and interpret results.

Regarding the time period, Aznar et al. (2016) advises against making forecasts for periods longer than ten years. The author suggests that the most accurate and common forecasts typically cover an interval of four to eight years. Following these recommendations, it is possible to establish a valuation period in two phases using the following mathematical expression:

$$V_0 = \frac{CF_1}{1+k} + \frac{CF_2}{(1+k)^2} + \frac{CF_3}{(1+k)^3} + \cdots + \frac{CF_n + TV_n}{(1+k)^n} \quad (1)$$

where V_0 represents the value of the company at the initial moment, CF_n represents the cash flow generated by the company in period n, TV_n represents the terminal value of the company in year n, and k represents the discount rate appropriate to the risk of the cash flows.

Assuming the continuity of the company, it is necessary to calculate its terminal value (value in perpetuity). For this calculation, a constant growth rate (g) must be applied to the cash flows after period n. The growth rate can be g (growth from the free cash flow for shareholders) or g_u (growth from free cash flow), depending on the type of cash flow being evaluated.

The terminal value is calculated using the following formula:

$$TV_n = \frac{CF_n \times (1+g)}{(k-g)} \quad (2)$$

where TV_n represents the terminal value of the company in period n, CF_n represents the cash flow generated by the company in period n, g is the sustainable growth rate appropriate to the cash flows used, and k represents the discount rate appropriate to the risk of the cash flows.

When valuing a company, two different approaches can be adopted. The first approach involves valuing the company as a whole, while the second focuses specifically on valuing equity (Damodaran, 2012; Steiger, 2010).

According to Neves (2002), various types of cash flows can be used in the valuation process, as outlined in Table 1:

Table 1. Discounted cash flow valuation methods

Evaluation perspective	Shareholders	Company		
Relevant cash flow	Free cash flow to equity	Free cash flow	Free cash flow	Tax savings
Discount rate	Cost of equity (k_e)	Weighted average cost of capital (WACC)	Cost of economic capital (k_{eu})	Market interest rate (k_d)
Method	Equity method	Average cost method	VALA method	

Source: Adapted from Neves (2002)

2.2.1.2 Limitations of DCF

While the discounted cash flow (DCF) model is widely regarded as the most effective method for determining a company's value, it also has several limitations. Damodaran (2012) identifies the following:

(a) **Companies with complicated financial situations**. For valuation purposes, future cash flows (free cash flows) are generally expected to be positive. However, companies in financial distress often generate negative cash flows. In such cases, it becomes challenging to evaluate these companies effectively, as the DCF method relies on positive cash flows. Negative cash flows will lead to a negative company valuation.

(b) **Companies with cyclical results**. Some companies experience cyclical performance that correlates with the economy. During periods of economic growth, these companies perform well, but during economic downturns or recessions, they may show reduced profits or negative cash flows. In such situations, the company may appear to be at risk of bankruptcy. Accurately valuing these companies requires incorporating forecasts of the broader economy's trajectory.

(c) **Companies with unused assets**. The DCF method captures the value of assets that generate cash flows. However, many companies possess unused assets that do not produce cash flows. As a result, the valuation may be incomplete or inaccurate, as the value of these unused assets is not reflected in the final estimation.

Pacheco et al. (2021) further note that accountants may arrive at differing conclusions regarding the amounts and timing of future cash flows. Additionally, there is often a lack of consensus about the appropriate adjustments to address uncertainty and risk.

French and Gabrielli (2005) highlight another limitation, suggesting that undiscounted measures provide more reliable values than present value calculations. They argue that the latter introduces inaccuracies due to the reliance on estimates.

2.2.1.3 Economic value added (EVA)

According to Pacheco et al. (2021), economic value added (EVA) is a simpler and less demanding method of estimating value compared to other approaches, such as the discounted cash flow (DCF) method, as it requires less financial information. EVA measures the monetary value generated by a company through its existing investments and is calculated as the product of the excess return created by an investment compared to the capital invested in that same investment. This valuation model helps managers determine whether their decisions are creating or destroying value (Vieito & Maquieira, 2013).

Sharma and Kumar (2010) describe EVA as an estimate of true economic profit, reflecting the difference between earnings obtained and the minimum required rate of return that creditors and shareholders could achieve by investing in securities with a similar level of risk. Value creation occurs when the return on invested capital exceeds the weighted average cost of capital (WACC). Conversely, when the return on invested capital is lower than the WACC, value destruction occurs (Pacheco et al., 2021; Vieito & Maquieira, 2013).

According to the aforementioned authors, EVA is calculated using the following formula:

$$EVA = (Return\, on\, capital\, invested - Cost\, of\, capital) \times Capital\, invested \quad (3)$$

Damodaran (2012) explains that the valuation model can be understood as a combination of three components: the capital invested in existing assets, the present value of the economic value added (*EVA*) generated by these existing assets, and the expected present value of the economic value added by future investments. This is expressed as:

$$V_0 = Capital invested_{assets\, in\, place} + \sum_{t=1}^{t=\infty} \frac{EVA_{t,assets\, in\, place}}{(1+k)^t} + \sum_{t=1}^{t=\infty} \frac{EVA_{t,future\, projects}}{(1+k)^t} \quad (4)$$

where: V_0 represents the value of the company at the initial moment and k represents the cost of capital.

2.3 Forecasting cash flows

2.3.1 Using the historical growth rate in the forecast period

According to Aznar et al. (2016), the growth rate to be applied during a forecast period can be derived from analyzing the historical behavior of different variables, with turnover being the most commonly accepted. The authors justify the choice of this variable because it is the least influenced by accounting criteria.

When analyzing historical growth rates, it is crucial to understand two approaches to estimating growth rates for the forecast period: the geometric average rate and the arithmetic average rate. According to Damodaran (2012), the arithmetic average rate is a simple average of past growth rates, while the geometric average rate accounts for the cumulative effect of growth over multiple periods. These rates, as explained by Neves (2002), are represented as follows:

$$Arithmetic\,average:\; g_a = \frac{\sum_{i=0}^{n} \frac{Earnings_{i+1} + Earnings_i}{Earnings_i}}{n} \quad (5)$$

where $Earnings_{i+1}$ represents the value of income in year $i+1$, $Earnings_i$ represents the value of income in year i, and n represents the number of years under analysis.

$$Geometric\,average:\; g_g = \left(\frac{Earnings_n}{Earnings_0}\right)^{\frac{1}{n}} - 1 \quad (6)$$

where $Earnings_n$ represents the value of income in the last year under review, $Earnings_0$ represents the value of income in the first year under analysis, and n represents the number of years under analysis.

Neves (2002) points out that the arithmetic and geometric growth rates can differ significantly if a company has highly volatile results. The author identifies specific situations in which it is impossible to apply one rate or the other: (a) when the variable under analysis is net profit and it is negative, applying the arithmetic mean is not meaningful; (b) when the variable under analysis is dividends and they are zero, it is impossible to calculate the growth rate and, consequently, the respective arithmetic mean; and (c) when using the geometric mean, if the variable under analysis is net profit, the first net profit cannot be zero or negative, as this would make it impossible to calculate the growth rate. Given these limitations, the author recommends using turnover as the target variable for analysis.

Similarly, Damodaran (2012) concludes that the geometric growth rate is the most reliable measure of a company's true historical growth, particularly when year-on-year growth is irregular.

2.3.2 Residual period growth rate

In addition to the market's impact on a company's business growth, it is essential to evaluate the company's financial capacity to sustain this growth. When estimating the growth rate in the residual period, the sustainable growth rate must be considered, assuming no additional funding will be required from shareholders to support the company's growth (Neves, 2002).

The sustainable growth rate is the rate used to calculate a company's residual value and must align with the return on equity (ROE) and the reinvestment rate. Under the equity method, the sustainable growth rate (g) can be calculated as follows (Neves, 2002):

$$g = ROE \times (1 - d) \quad (7)$$

where g represents the sustainable growth rate, ROE represents the return on equity, d represents the distribution rate, and (1-d) represents the reinvestment rate.

Damodaran (2012) emphasizes the need to adjust the sustainable growth rate, as the earnings retention rate assumes all undistributed earnings will be reinvested in the company. The reinvestment rate is calculated as follows:

$$(1-d) = \frac{CAPEX + CIWC - Depreciation - (New\,debt\,issued - Debt\,repaid)}{Net\,Income} \quad (8)$$

where *CAPEX* represents the capital expenditures and *CIWC* represents change in working capital.

If the average cost method is used, the sustainable growth rate (g_u) can be calculated in two ways, as outlined by Neves (2002):

$$g_u = \frac{EBIT \times (1-t)}{D+E} \times (1-d)$$
$$g = g_u \left(1 + \frac{D}{E}\right) <=> g_u = \frac{g}{(1+\frac{D}{E})} \ (9)$$

where g_u represents the sustainable growth rate of operating income, g represents the sustainable growth rate of net profit, *EBIT* represents the earnings before interest and taxes, (1-d) represents the reinvestment rate, D represents debt, and E represents equity.

Since it is unlikely that a company can sustain perpetual growth at a rate higher than the growth rate of the economy or the sector (g_n) in which it operates, it is essential to use the lower rate when defining the sustainable growth rate (Neves, 2002). To ensure that the residual value does not excessively impact the company's final value, a relatively low growth rate is recommended (Aznar et al., 2016).

The economy's growth rate can be calculated using the following formula:

$$g_n = (1 + GDP\ growth\ rate) \times (1 + inflation\ rate) - 1 \ (10)$$

where g_n represents the growth rate of the economy and *GDP* represents gross domestic product.

2.3.3 Determining cash flows through historical results

The Financial Accounting Standards Board (FASB, 1978) identifies historical results as a key element for forecasting cash flows. Consequently, numerous studies have empirically examined this proposition, specifically the determination of future cash flows based on historical results. Notable studies include those by Barth et al. (2001), Al-Attar and Hussain (2004), Kim and Kross (2005), Takhtaei and Karimi (2013) and Nguyen and Nguyen (2020).

Nguyen and Nguyen (2020) and Noury et al. (2020) suggest that the explanatory variables to be included in forecasting models should include net income lagged by one year, two years, three years, or more. According to these authors, cash flows from operations can be estimated using the following model:

$$CFO_t = \beta_0 + \beta_1 \times Earnings_{t-1} + \beta_2 \times Earnings_{t-2} + ... + \beta_n \times Earnings_{t-n} + \varepsilon \ (11)$$

where CFO_t represents the cash flow from operations at time t; β_0 represents the model constant; $\beta_1,...,\beta_n$ represent the regression coefficients between each of the lagged net results and the cash flow at time t; $Earnings_{t-1},..., Earnings_{n-1}$ represent the net income lagged by one year, two years, and so forth; and ε represents the regression error.

Noury et al. (2020), analyzing a sample of 61 French non-financial listed companies from 1999 to 2016, concluded—using the ordinary least squares (OLS) model—that net income lagged by three periods demonstrates some predictive ability for cash flows.

Meanwhile, Nguyen and Nguyen (2020) analyzed data from 242 non-financial companies listed on the Ho Chi Minh Stock Exchange (HOSE) between 2009 and 2018. They employed three models to estimate the panel data: the ordinary least squares (OLS) model, the random effects model (REM), and the fixed effects model (FEM). The results demonstrated that lagged earnings have a significant ability to predict cash flows from operations in future years. Moreover, the authors noted that the model's predictive capacity improves as additional lag years are incorporated.

2.4 Cost of capital

2.4.1 Cost of equity models

To achieve an optimal financial structure, it is crucial to evaluate the costs associated with financing capital investments. The ideal financial structure minimizes a company's financial expenses while maximizing its value. In this context, it is necessary to calculate the cost of equity (k_e), the cost of debt capital (k_d), and the weighted average cost of capital (WACC), which results from the simultaneous use of these financing sources (Pacheco et al., 2021).

Both creditors and shareholders expect compensation for the opportunity cost of investing their money in a specific business instead of other ventures with equivalent risk (Copeland et al., 2000; Exley & Smith, 2006; Neves, 2002). Opportunity cost encompasses both profitability and risk and serves as the discount rate for future financial flows in various valuation models (Pacheco et al., 2021). This rate must be consistent with the cash flow being calculated (Damodaran, 2012).

According to Pratt and Grabowski (2014), the cost of capital is arguably one of the most critical concepts in finance. A correct estimate of this cost allows expected income streams to be converted into present value estimates, enabling investors to compare different investment opportunities and select the one offering the highest return (McConaughy, 1999).

Neves (2002) highlights that debt and equity values should be expressed at market prices. However, for privately held companies, market prices are unavailable, unlike publicly held companies. In such cases, book values are used instead, though these do not convey the same type of information.

Furthermore, calculating the WACC for privately held companies is more challenging due to the difficulty of estimating the values of equity and debt capital and their respective costs, owing to the lack of publicly available information.

2.4.1.1 The CAPM model

The capital asset pricing model (CAPM) was developed by Sharpe in 1964 and Lintner in 1965, building on concepts introduced by Markowitz in 1952 in modern portfolio theory. This theory emphasizes the importance of efficiency and diversification in investment portfolios. CAPM proposes a linear relationship between the return required by shareholders and risk (Pacheco et al., 2021; Torrez et al., 2006).

According to Rossi (2016), CAPM remains widely used to estimate the cost of capital for companies and assess the performance of investment portfolios. Under CAPM, the cost of equity (k_e) required by a company's managers represents the return investors expect given the specific risk associated with the investment (Neves, 2002; Pacheco et al., 2021). This relationship is expressed as follows:

$$E(r_e) = r_f + \beta_e \times [E(r_M) - r_f] \quad (12)$$

where $E(r_e)$ represents the return required by shareholders, r_f represents the risk-free interest rate, β_e represents the risk index of the financial asset, $E(r_M)$ represents the expected market return, and $E(r_M)$-r_f represents the market risk premium.

According to Neves (2002) and Pacheco et al. (2021), CAPM relies on several assumptions: (a) market efficiency, (b) stability over time in the relationship between risk and return, and (c) the premise that investors are risk-averse, demanding higher returns for higher risk levels.

Black et al. (1972) further argue that all investors share a homogeneous view of the returns and risks of financial assets and have the ability to lend and borrow at a given risk-free rate.

Petersen et al. (2006) and Damodaran (2012) highlight that a fundamental assumption for applying CAPM is that investors are rational and well-diversified, allowing them to avoid exposure to specific risks. However, this condition applies primarily to publicly traded companies, as diversification strategies are typically unavailable to privately held companies. In most cases, the owner of a private company acts as the sole investor, concentrating all their wealth in the business (Petersen et al., 2006; Rojo-Ramírez, 2014).

Damodaran (2012) also emphasizes the absence of a market beta for private companies due to the lack of historical stock prices, which makes it impossible to calculate the beta directly. Consequently, alternative methods for estimating a market beta must be employed.

2.4.1.1.1 Risk-free rate

According to Pratt and Grabowski (2014), the risk-free rate represents the return available, as of the valuation date, on a security that the market considers free from default risk.

As Neves (2002) notes, the yield to maturity (YTM) of a financial instrument is commonly used to estimate the risk-free rate, depending on the instrument's duration. For short-term durations, the interest rate on treasury bills is applied, while for longer terms, the interest rate on treasury bonds is typically used (Pacheco et al., 2021). These instruments are preferred because they are issued by governments, making the risk of default nearly nonexistent. A default would imply a country's bankruptcy, which is considered an extremely unlikely scenario (Vieito & Maquieira, 2013).

It is a common practice for appraisers to use the YTM of treasury bonds (TB) with maturities of approximately ten years (Neves, 2002).

Damodaran (2023) emphasizes that the risk-free rate must be entirely free of risk, which is not the case when country default risk is factored in. Therefore, interest rates on bonds issued by governments in emerging markets, denominated in their local currencies, should not be used as a risk-free rate. Instead, Damodaran argues that the risk-free rate should align with the currency in which the cash flows are defined. For instance, if cash flows are denominated in US dollars, the appropriate risk-free rate would be the interest rate on US treasury bonds.

Damodaran (2023) outlines three approaches for identifying the risk-free rate when accounting for country default risk. Among these, the approach yielding the lowest risk-free interest rate should be selected:

1. If the country of the currency being analyzed issues US dollar-denominated treasury bonds, the YTM of these ten-year treasury bonds should be used. This YTM is then compared with the YTM of US treasury bonds for the same evaluation period to determine the country default risk.
2. Use the Credit Default Swap (CDS) spread as a measure of country default risk.
3. Use the Sovereign Default Spread (SDS), derived by converting the sovereign rating assigned by a credit rating agency, as a measure of country default risk.

2.4.1.1.2 Market risk premium

According to Pacheco et al. (2021), the market risk premium is the difference between the average rate of return on market shares and the average rate of return on risk-free assets. It is typically derived from historical data.

To ensure consistency, the period used to define the risk-free rate for calculating the market risk premium should align with the period used to determine the return required by shareholders. In this regard, Aznar et al. (2016) suggest using the YTM of ten-year government bonds as a reference.

Damodaran (2023) identifies three approaches for determining the Equity Risk Premium (ERP) of a given country:

1. **Historical market risk premium**: Determine the country's historical market risk premium by analyzing the historical returns of risky stocks and comparing them to the returns of risk-free investments. This historical risk premium is then used as the expected future risk premium.
2. **Modified historical risk premium**: Use the risk premium of a mature market, such as the US, and add an additional premium that reflects the inherent risk of the country being analyzed.
3. **Implicit market risk premium**: Estimate a future risk premium based on the price of the assets being traded, thereby deriving the implied equity risk premium.

Under the second approach, Damodaran (2023) notes that countries with AAA ratings from credit rating agencies such as Standard & Poor's are considered free of country risk and do not require an adjustment. For emerging markets, however, an additional country risk premium must be added to the risk premium of a mature market to account for country-specific risks. This relationship can be expressed as:

$$ERP_X = ERP_M + CRP_X \quad (13)$$

where ERP_X represents the equity risk premium of the country being valued (X), ERP_M represents the equity risk premium of a mature market, and CRP_X represents the country risk premium of the country being valued (X).

According to Damodaran (2023), one way to determine the country risk premium is as follows:

$$Country\ risk\ premium = Country\ default\ spread \times \left(\frac{\sigma_{Equity}}{\sigma_{Country\ Bond}}\right) \quad (14)$$

where σ_{Equity} represents the volatility of the stock market of the country being valued and $\sigma_{Country\ Bond}$ represents the volatility of the bond market of the country being valued.

Damodaran (2023) also outlines three approaches to calculating the cost of equity to reflect varying levels of a firm's exposure to country risk:

1. **Identical exposure to country risk and other market risks**:

$$E(R_e) = r_f + \beta \times ERP_X \quad (15)$$

where $E(R_e)$ represents the expected return on equity, r_f represents the risk-free rate, β represents the beta of the company, and ERP_X represents the equity risk premium of the country being valued.

2. **Uniform exposure to country risk across all companies**:

$$E(R_e) = r_f + CRP_X + \beta \times ERP_M \quad (16)$$

where $E(R_e)$ represents the expected return on equity, r_f represents the risk-free rate, β represents the beta of the company, ERP_M represents the equity risk premium of a mature market, and CRP_X represents the equity risk premium of the country being valued.

3. **Country risk as an isolated risk factor**:

In this approach, country risk is treated as a separate risk factor, with companies having different levels of exposure based on their sales to third countries:

$$E(R_e) = r_f + Lambda + CRP_X + \beta \times ERP_M$$

where

$$Lambda = \frac{\%\ company\text{`}s\ domestic\ revenues}{\%\ average\ domestic\ revenues\ of\ companies} \quad (17)$$

and where $E(R_e)$ represents the expected return on equity, r_f represents the risk-free rate, *Lambda* represents a company's level of exposure to country risk, β represents the beta of the company, ERP_M represents the equity risk premium of a mature market, and CRP_X represents the equity risk premium of the country being valued.

2.4.1.1.3 Beta

Beta is a measure of risk that reflects the volatility of a company's share returns relative to the average return of the overall market, representing the specific risk that an investment adds to a portfolio (Cañadas & Rojo Ramirez, 2011).

The main distinction in calculating betas for privately held companies versus publicly held companies lies in the availability of historical market data. According to Damodaran (2012), in the absence of historical data, private companies can estimate their beta using one of four methods: (a) accounting beta, (b) fundamental beta, (c) bottom-up beta, and (d) total beta. In contrast, publicly traded companies, which have access to historical data, can use these methods as well as the historical market beta method (Damodaran, 2012). Fernández (2015) also identifies a fifth approach called qualitative beta.

For the **historical market beta**, the beta of an investment is estimated by regressing the returns of the investment against the returns of a market index. This regression is based on the Sharpe (1964) market model:

$$r_e = a + \beta \times R_M \quad (18)$$

where r_e represents the return on the share, a represents the intercept of the linear regression, β represents the stock's beta, and R_M represents the return on the market index.

In the case of the **accounting beta**, Damodaran (2012) suggests using accounting earnings information instead of market prices to estimate beta. This is done by performing a linear regression of changes in a private company's accounting earnings against changes in the earnings of a stock index, such as the S&P 500 or the PSI (Portuguese Stock Index). However, the author highlights two significant limitations to this approach: (a) earnings are typically calculated only once a year, resulting in a limited number of observations for the regression, and (b) accounting earnings can be smoothed or manipulated, potentially compromising their credibility.

Almisher and Kish (2000) argue that accounting beta can serve as a substitute for market beta in situations where market data is unavailable. This relationship can be expressed as follows:

$$\Delta\ Earnings_{Private\ Company} = a + \beta \times \Delta\ Earnings_{S\&P\ 500} \quad (19)$$

where β represents the company's accounting beta and a represents the intercept of the linear regression.

Regarding the **fundamental beta**, Beaver et al. (1970) established a relationship between the betas of listed companies and observable variables, such as earnings growth. These authors identified six variables that influence beta: dividend payments, asset growth, degree of indebtedness, asset size, variation in earnings, and the accounting beta. Damodaran (2012) asserts that since these variables can be collected for private companies, it is possible to estimate a fundamental beta for such businesses. However, the author notes a significant limitation: regressions using this method often result in low coefficients of determination (R^2), which leads to forecasts with high standard errors. This imprecision, in turn, affects the accuracy of corporate risk estimates.

As for the **bottom-up beta**, Bowman and Bush (2007) highlight that the most common approach to estimating the beta of a private company is to use the average betas of comparable publicly listed companies as a proxy. This method emphasizes using an industry beta rather than relying on the beta of a single company or a small subset of companies. The authors argue that using an industry beta provides a more reliable estimate of a private company's beta due to its higher comparability.

Damodaran (2012) outlines five steps for applying the bottom-up beta method:

1. Identify the sector or sectors in which the company operates.
2. Identify other comparable publicly traded companies in each sector and obtain their regression betas to calculate an average for these companies' betas.
3. Estimate the unlevered betas (β_{eu}) of the comparable companies.
4. Estimate the unlevered beta (β_{eu}) of the company under analysis by considering the weighted average of the unlevered betas in the business areas where the company operates.
5. Leverage the beta obtained in the previous step through the company's financing structure to calculate the leveraged beta (β_e) of the company under analysis.

Beneda (2003) provides the formulas for leveraging and de-leveraging beta, assuming that debt does not carry market risk (i.e., the debt beta (β_d) is zero):

$$\beta_e = \beta_{eu} \times \left[1 + \left(\frac{D}{E} \times (1-t)\right)\right] \quad (20)$$

where β_e represents the leveraged beta, β_{eu} represents the unlevered beta, D represents the value of debt, E represents the value of equity, and t represents the tax rate.

For Damodaran (2012), when the debt beta (β_d) carries market risk, it should be considered. While the author argues this approach is more realistic, estimating the debt beta can be challenging. When accounting for the debt beta, the formulas for leveraging and de-leveraging beta are as follows:

$$\beta_e = \beta_{eu} + (\beta_{eu} - \beta_d) \times \left[\left(\frac{D}{E} \times (1-t)\right)\right] \quad (21)$$

where β_e, represents the leveraged beta, β_{eu} represents the unlevered beta, β_d represents the debt beta, D represents the value of debt, E represents the value of equity, and t represents the tax rate.

Damodaran (2012) emphasizes the lack of diversification in privately held companies, where the company owner is often the sole investor and thus lacks opportunities to diversify their investments. As a result, if the owner concentrates all their wealth in their own business and is not adequately diversified, they are exposed to total risk rather than just market risk (measured by beta). To address this issue, the author suggests determining the **total beta** by adjusting the market beta to reflect total risk rather than market risk. This adjustment accounts for the correlation between the company and the market:

$$Total\ Beta = \frac{Market\ Beta}{Correlation\ with\ Market} \quad (22)$$

The lower the correlation between the company's return and the market return, the higher the total beta (Damodaran, 2012).

Regarding **qualitative beta**, Fernández (2015) argues that companies should have a beta and market risk premium tailored to their specific characteristics, opposing the assumption in the CAPM model that all investors should use the same beta and market risk premium. The author suggests that these variables be calculated based on common sense and consideration of factors such as the company's unique attributes, the surrounding industry, and the national economy.

To achieve this, Fernández (2015) proposes the MASCOFLAPEC method, which accounts for the following factors: Management, Assets, Strategy, Country risk, Operating leverage, Financial leverage, Liquidity of investment, Access to sources of funds, Partners, Exposure to other risks, and Cash flow stability. Each parameter is assigned a score from 1 to 5 based on its level of risk, with a weighted score applied to reflect its significance in estimating beta.

2.4.1.2 Cost of equity models based on the CAPM

2.4.1.2.1 Fama and French model

Fama and French (1993) developed a three-factor model that extends the CAPM framework. According to Brealey et al. (2020), this model incorporates market risk, company size, and the relationship between book value and market value. Neves (2002) explains that the return on assets can be attributed to their sensitivity to three factors: (a) the profitability of the market, measured by a market risk indicator (β); (b) the difference in profitability between small and large companies, known as SMB (Small minus Big); and (c) the difference in profitability between companies with high and low book-to-market ratios, known as HML (High minus Low).

According to Fama and French (1993), the model is represented as follows:

$$R_{i,t} - r_{f,t} = \alpha_i + \beta_{i,m} \times [R_{m,t} - r_{f,t}] + \beta_{i,smb} \times SMB_t + \beta_{i,hml} \times HML_t + \varepsilon_{i,t} \quad (23)$$

where $R_{i,t}$ represents the rate of return on asset i in period t, $r_{f,t}$ represents the risk-free rate in period t, α_i represents the excess return of asset i not explained by the risk factors in the model, $R_{m,t}$ - $r_{f,t}$ represents the market risk premium in period t, *SMB* represents the difference in profitability between small and large companies, *HML* represents the difference in profitability between companies with high and low book-to-market equity ratios, $\beta_{i,m}$ represents the sensitivity of asset i to market risk, $\beta_{i,smb}$ represents the sensitivity of asset i to company size, $\beta_{i,hml}$ represents the sensitivity of asset i to the book-to-market ratio, and ε_{it} represents the error term of asset i in period t.

Brealey et al. (2020) note that, in practice, the Fama and French model is less commonly used than the CAPM, primarily because it requires estimating and interpreting three different betas, along with three risk premiums. In contrast, the CAPM uses a simpler framework, relying on just one beta and one market risk premium.

2.4.1.2.2 Arbitrage pricing theory model

Ross (1976) developed an alternative model for calculating the cost of capital, known as the arbitrage pricing theory (APT) model, which posits that the rate of return depends on multiple independent factors. Unlike the classic CAPM model, which relies on a single systematic risk factor, the APT model incorporates several betas, each measuring the sensitivity of a company’s stock return to a specific underlying economic factor (Pratt & Grabowski, 2014; Torrez et al., 2006).

According to Pratt and Grabowski (2014), the model is represented as follows:

$$E(R_i) = r_f + (\beta_{i1} \times RP_1) + (\beta_{i2} \times RP_2) + ... + (\beta_{in} \times RP_n) + \varepsilon \quad (24)$$

where $E(R_i)$ represents the expected return for asset i, r_f represents the risk-free rate, β_{in} represents the sensitivity of the return on asset i to factor n, RP_n represents the risk premium associated with risk factor n, and ε represents the error term.

Torrez et al. (2006) criticize the model for its lack of clear guidelines in financial theory regarding which variables (factors) should be included or excluded. In response, Pratt and Grabowski (2014) suggest the following risk factors for consideration: interest rate risk, inflation rate risk, yield spread, and risks associated with the business outlook.

2.4.1.2.3 Three-component model

The three-component model, proposed by the Spanish Association of Accounting and Business Administration (AECA) in 2005 and named by Rojo-Ramírez (2014), was developed as an alternative to the CAPM for calculating the cost of capital in private companies.

Rojo-Ramirez et al. (2012) argue that the CAPM model is unsuitable for private companies due to the absence of a specific market beta, which makes the conventional application of CAPM for estimating the cost of capital inaccurate. Furthermore, the authors emphasize the importance of considering the nature of the investor when determining the cost of capital for private entities.

Thus, the aforementioned authors distinguish investors into two categories:

1. **Purely financial investor (PFI)**: A purely financial investor (PFI) is an individual who uses the market to

diversify and liquidate their portfolio, aiming to maintain a proportional portfolio of securities such that the beta equals 1. A PFI seeks to achieve the minimum return by combining the risk-free interest rate and the market risk premium. Accordingly, the cost of capital for a PFI is calculated as if it were for a public company:

$$k_e = r_f + (r_M - r_f) = r_f + P_M \ (25)$$

where k_e represents the cost of equity, r_f represents the risk-free rate, r_M represents the expected market return, and P_M represents the market risk premium.

2. **Economic risk investor (ERI)**: An economic risk investor (ERI) invests all their wealth in a single company, without adopting any diversification strategy, and faces the illiquidity of their investment. This type of investor is typical in private companies and requires an additional risk premium, referred to as the company-specific risk premium (P_e). Since an ERI assumes the specific risk of an undiversified economic activity with illiquidity, they demand a higher minimum return than a PFI. The cost of capital for an ERI is calculated as if it were for a private company:

$$k_e = r_f + P_M + P_e \ (26)$$

where k_e represents the cost of equity, r_f represents the risk-free rate, P_M represents the market risk premium, and P_e represents the specific risk of the company.

According to Rojo-Ramírez et al. (2012), consider an investor who allocates all their wealth to a financial asset replicating a market index, with a return of R_M and standard deviation σ_M. Simultaneously, this investor takes on debt at a risk-free interest rate (R_f) and invests the borrowed amount in an economic activity with a return of R_e and standard deviation σ_e. In this scenario, an economic risk investor (ERI) can be viewed as managing a mixed investment portfolio, where the total return is the combination of two investments: a risk-free portfolio and a risky portfolio, adjusted for the costs associated with the debt. This relationship is expressed as:

$$k_e = R_f + P_M + P_M \times \frac{\sigma_e}{\sigma_M}$$
$$\text{where the specific risk is } P_e = P_M \times \frac{\sigma_e}{\sigma_M} \text{ and } \beta_T = \frac{\sigma_e}{\sigma_M} \ (27)$$

where k_e represents the cost of equity, r_f represents the risk-free rate, P_M represents the market risk premium, and σ_e / σ_M represents the coefficient of variability of the company's risk in relation to the market.

The authors argue that this result aligns with the concept that a company's risk is proportional to its market risk. Therefore, the best way to calculate the specific risk premium (P_e) is to adjust the market risk premium (P_M) using the coefficient of variability of the company's risk relative to the market, referred to as the total beta (β_T).

Rojo-Ramírez (2014) referred to the model as the "three-component" model because it is based on three fundamental principles: (a) the investor must ensure that the return obtained is higher than the risk-free interest rate; (b) a non-diversified investor must achieve at least the same return as a diversified investor; and (c) the return required by a non-diversified investor must be adjusted to account for the specific risk assumed.

The author emphasizes the importance of using an appropriate discount rate when applying the discounted cash flow method to calculate the value of a company. An incorrect discount rate could lead to either an undervaluation or overvaluation of the company's value.

2.4.1.2.4 Cost of equity model for family businesses

De Visscher et al. (1995), as cited in McConaughy (1999), proposed an innovative model, based on the CAPM, to calculate the cost of capital for family businesses. The model is represented as follows:

$$k_e = [r_f + \beta(R_M - r_f)] \times (1 + IP) \times (1 - FE) \ (28)$$

where k_e represents the cost of equity, r_f represents the risk-free rate, β represents the market beta, R_M represents the market return, *IP* represents the illiquidity premium, and *FE* represents the family effect.

According to McConaughy (1999), the term (1+ *IP*) reflects the impact of the illiquidity premium on the shares of private companies, resulting in an increase in the cost of capital. This value ranges between 0 and 1. The term (1- *FE*) represents the influence of the family on the business, also varying between 0 and 1, where 0 signifies a highly litigious family and 1 represents a family that is highly involved and committed to the business.

McConaughy (1999) cautions that the family effect could pose challenges, as a value of 1 would result in a cost of equity of 0%. Such a scenario contradicts financial theory, as it is unreasonable for a financial institution to lend money at a 0% cost without receiving any financial return. Furthermore, no rational investor would accept a return lower than what banks offer on time deposits.

2.4.1.3 Cost of equity models not based on the CAPM

According to Pereiro (2001), existing empirical evidence has failed to validate the applicability of the CAPM model in emerging markets. Erb et al. (1996) argue that while the beta-based approach is effective in developed markets, it produces more uncertain results in emerging markets. Similarly, Harvey (1995) and Estrada (2000) conclude that emerging markets are highly volatile, and there is no clear correlation between expected returns and betas measured against the global market.

Due to these challenges, alternative models that incorporate risk measures beyond the beta used in CAPM were necessary (Pereiro, 2001). Two prominent alternatives are the model proposed by Javier Estrada in 2000, referred to as *The Estrada Model*, and the model introduced by Claude Erb, Campbell Harvey, and Tadas Viskanta in 1996, known as *The Erb-Harvey-Viskanta Model*.

2.4.1.3.1 The Estrada model

Estrada (2000) argues that total risk, specific risk, and certain measures of downside risk are significantly correlated with stock returns in emerging markets. To address this, the author proposes a model for estimating the cost of equity capital in emerging markets based on the semi-deviation from the mean, a well-recognized measure of downside risk.

According to Estrada (2000), this is a straightforward model that can be implemented as easily as the CAPM. Grounded in modern portfolio theory, it can be applied at both the company and market levels without relying on subjective risk measures. The model can also be adapted to any chosen market index, directly addressing the downside risk investors aim to avoid.

The model is designed from the perspective of a US investor with an internationally diversified portfolio. In this framework, the risk-free rate compensates the investor for the expected loss in the US dollar transaction, while the risk premium rewards the investor for investing in a global portfolio. The model is expressed as:

$$k_e = r_{f,US} + (R_{M,W} - r_{f,W}) \times RM_i \ (29)$$

where k_e represents the cost of equity, $r_{f,US}$ represents the risk-free rate in the United States, $R_{M,G}$ represents the return of the global market, $r_{f,G}$ represents the global risk-free rate, and RM_i represents a measure of market risk i.

Estrada (2000) further proposes estimating the returns of an emerging market using a downside risk measure. Specifically, RM_i is defined as the ratio of the semi-standard deviation of returns relative to the mean of market i to the semi-standard deviation of returns relative to the mean of the global market. The semi-standard deviation of returns for any market index, denoted as $B(\Sigma_B)$, is calculated as follows:

$$\sum B = \sqrt{\left(\frac{1}{T}\right) \times \sum_{t=1}^{T} (R_t - B)^2} \ , \ \ for\ all\ R_t < B \ (30)$$

where R_t represents the returns, t represents the moment in time, and T represents the total number of observations.

2.4.1.3.2 The Erb-Harvey-Viskanta (EHV) model

Erb et al. (1996) argue that estimating risk in emerging markets using beta is often impractical because many emerging markets lack a functioning stock market. To address this, the authors propose a model for economies without a stock market, based on the country's credit rating.

The country credit rating (CCR) is obtained biannually from *Institutional Investor* magazine in March and September. Countries are assigned a score ranging from 0 to 100, with 100 representing the lowest risk of default (Erb et al., 1996). The model is represented as follows:

$$R_{i,t+1} = \gamma_0 + \gamma_1 \ln(CCR_{it}) + \epsilon_{i,t+1} \ (31)$$

where R represents the six-month return in dollars for country i; CCR represents the country credit rating; t

represents time, measured in six-month periods; γ represents the risk premium; and ε represents the regression error.

According to Pereiro (2001), the EHV model is based on a risk measure independent of the stock market, using the country's debt rating to determine the cost of equity capital. This measure integrates typical variables of country risk, such as political risk, exchange rate risk, and inflation risk.

2.4.1.3.3 The build-up model

The build-up model breaks down the cost of equity into several components, assigning a percentage to each (Boudreaux et al., 2012). Unlike models that rely on data from publicly listed companies, the build-up approach is commonly used for small and medium-sized enterprises (SMEs) and was developed by Professor Roger Ibbotson, making it widely known as "Ibbotson's Build-Up Method". This model calculates the cost of equity by combining both systematic and unsystematic risks.

According to Ballwieser and Wieser (2010) and Boudreaux et al. (2012), the cost of equity is calculated using the following expression:

$$k_e = r_f + ERP + RP_{size} + RP_{industry} + RP_{company} \quad (32)$$

where k_e represents the cost of equity, r_f represents the risk-free rate, *ERP* represents the equity risk premium, RP_{size} represents the risk premium associated with the size of the company, $RP_{industry}$ represents the risk premium associated with the company's industry, and $RP_{company}$ represents the specific risk premium associated with the company.

Ballwieser and Wieser (2010) note that while the first three variables in Equation (32) can be quantified reliably due to the availability of robust data, the remaining components are more challenging to measure, especially for SMEs. Systematic risk is represented by the risk-free rate and the market risk premium, whereas unsystematic risk consists of the industry risk premium, the size risk premium, and the company-specific risk premium.

According to Pratt and Grabowski (2014), the size risk premium can be sourced from *Morningstar's Ibbotson Stocks, Bonds, Bills and Inflation (SBBI) Yearbooks* or the *Duff & Phelps Risk Premium Report Size Study*. For the industry risk premium, Ibbotson developed a methodology that uses a full information beta, derived from data on companies within the same industry, to capture industry-specific risk and characteristics (Tang, 2023).

Regarding the company-specific risk premium, Pratt and Grabowski (2014) suggest evaluating factors such as: (a) the smaller size of companies in the lowest size risk premium group, (b) the risk of the sector, (c) the volatility of returns, and (d) other unique factors inherent to the company. Companies smaller than those included in the smallest group (10z decile) provided by Duff & Phelps or Morningstar must undergo additional adjustments to the size premium, reflecting the extra risk associated with their size.

Ibbotson also provides a formula to calculate the industry risk premium:

$$RPI_i = (RI_i \times ERP) - ERP \quad (33)$$

where RPI_i represents the expected risk premium of industry *i*, RI_i represents the risk index of industry *i*, and *ERP* represents the equity risk premium.

2.4.2 Cost of debt

According to Damodaran (2012), there are two possible scenarios for calculating the cost of debt (k_d) for listed companies: (a) if the company has long-term bonds actively traded in the market (high liquidity), the cost of debt should be the yield to average maturity of all bond issues; and (b) if the company has listed bonds but with low transaction levels (low liquidity), the cost of debt should be estimated using the assigned credit rating (as such companies typically have a credit rating) and the corresponding default spread.

For privately held companies or those not assessed by rating agencies, Damodaran (2012) proposes two alternatives for estimating the cost of borrowed capital: (a) using the interest rate of the most recent loan obtained by the company; and (b) estimating a synthetic rating based on the interest coverage ratio, while considering the size of the company. The synthetic rating can be obtained from a table published annually by Damodaran, which is then used to estimate the default spread (credit risk spread). The interest coverage ratio is calculated as:

$$Interest\ Coverage\ Ratio = \frac{EBIT}{Interest\ Expenses} \quad (34)$$

In a developed market, the cost of debt is determined as:

$$k_d = r_f + company\ default\ spread \quad (35)$$

where k_d represents the cost of debt and r_f represents the risk-free rate.

In an emerging market, country risk must be added to reflect additional risks:

$$k_d = r_f + company\ default\ spread + country\ default\ spread \quad (36)$$

where k_d represents the cost of debt and r_f represents the risk-free rate.

Neves (2002) and Pacheco et al. (2021) note that in situations where sufficient information is unavailable, historical data can be used to calculate the average cost of borrowed capital. This cost is represented as the ratio of interest and similar expenses incurred to total liabilities:

$$k_d = \frac{Interest\ Expenses}{Book\ Value\ of\ Debt} \quad (37)$$

2.4.3 The weighted average cost of capital

The weighted average cost of capital (WACC) is determined by considering the company's financial autonomy and debt ratios, appropriately weighted by their respective costs (Pacheco et al., 2021; Steiger, 2010). Steiger (2010) emphasizes that WACC is one of the most critical inputs when applying the discounted cash flow method, as it serves as the rate used to discount future cash flows. Even a small change in WACC can lead to significant variations in the company's valuation.

According to Brealey et al. (2020), the WACC can be calculated using the following formula:

$$WACC = \frac{E}{E+D} \times k_e + \frac{D}{E+D} \times k_d(1-t) \quad (38)$$

where *WACC* represents the weighted average cost of capital, *E* represents the value of equity, *D* represents the value of debt, *E+D* represents the total value of the company's assets, k_e represents the cost of equity, k_d represents the cost debt, and *t* represents the income tax rate.

2.5 The value of a company in perpetuity

As previously mentioned, it is common to divide the evaluation period into two distinct phases: the forecast period, during which the company's future cash flows are projected, and the terminal (or residual) period, which assumes the indefinite continuity of the company.

According to Steiger (2010), the terminal value represents the present value of all future cash flows occurring after the forecast period. Nissim (2019) highlights the significance of the terminal value, noting that it often accounts for approximately 70% to 80% of the company's total valuation.

Damodaran (2012) identifies two distinct approaches to a company's valuation outcome. One approach assumes the company will continue generating cash flows indefinitely, while the other adopts a liquidation approach, in which the company ceases operations and its assets are sold at a specific point in time. Under the continuity approach, it is unrealistic to assume that a company can sustain a high growth rate indefinitely. Over time, growth rates will inevitably decline and may eventually align with or fall below the growth rate of the economy or sector in which the company operates (Damodaran, 2012).

The value of a firm is calculated as:

$$Value\ of\ firm = \sum_{t=1}^{n} \frac{CF_t}{(1+k)^t} + \frac{TV_n}{(1+k)^n} \quad (39)$$

where CF_t represents the cash flow at time *t*, TV_n represents the terminal value at time *n*, and *k* represents the cost of capital.

According to Damodaran (2012), the terminal value can be estimated using three approaches:

1. **Liquidation value**: This method assumes that the company's assets will be liquidated at a specific point in the future (terminal year). The value of the accumulated assets is estimated based on what

potential buyers would be willing to pay at that time, thereby considering a finite life cycle for the company.

2. **Multiples approach**: This approach estimates the company's value in a future year by applying a multiple to a variable, such as sales for that year. It is based on relative valuation methods.
3. **Stable growth model**: This model assumes that cash flows beyond the terminal year will grow at a constant rate indefinitely, effectively in perpetuity. According to Nissim (2019), using the stable growth model to calculate the residual value is the most common practice.

The residual value can be calculated as follows:

$$Terminal\,Value_n = \frac{CF_{n+1}}{k - stable\;growth\;rate} \quad (40)$$

where CF_{n+1} represents the cash flow at time n+1 and k represents the cost of capital.

As mentioned earlier, the choice of the cash flow type and discount rate depends on whether the objective is to value the company as a whole or only its equity.

• **Valuing the company as a whole**: The terminal value is calculated as:

$$TV_n = \frac{FCF_{n+1}}{(WACC - g_u)} \quad (41)$$

where TV_n represents the terminal value in year n, FCF_{n+1} represents the free cash flow in year $n+1$, $WACC$ represents the weighted average cost of capital, and g_u represents the company's sustainable growth rate.

• **Valuing equity only**: The terminal value is calculated as:

$$TV_n = \frac{FCFE_{n+1}}{(k_e - g)} \quad (42)$$

where TV_n represents the terminal value in year n, $FCFE_{n+1}$ represents the free cash flow to equity in year $n+1$, k_e represents the cost of equity, and g represents the sustainable growth rate of equity.

3. Case study – company ABC

3.1 Presentation of company ABC

ABC operates in the manufacturing of metal molds, classified under economic activity code 25734. Founded in 1998, the company has experienced significant growth in turnover since its inception. This growth has been driven by continuous investments in the latest production technologies available in the market, enabling the company to meet the increasing demand for services in the molds industry.

The company operates in both domestic and international markets, maintaining a steadfast commitment to the quality of its services and the creation of value for its stakeholders.

As of 2022, ABC employed 38 individuals and is classified as a small company under Portuguese Decree-Law 372/2007. This classification is based on its employment size—more than ten but fewer than 50 employees—and its turnover, which is below ten million euros.

3.2 Valuation of company ABC

The valuation of ABC was conducted using the discounted cash flow (DCF) method, as recommended by Fernández (2023) and other scholars. To determine the company's overall value, the average cost model (Equation (39)) was applied, using free cash flow (FCF) as the relevant cash flow for the analysis.

The implementation of the average cost model involved dividing the valuation period into two distinct phases: the forecast period and the residual period. Following the guidance of Aznar et al. (2016), a five-year time horizon was chosen for the forecast period. This approach aligns with the *Norma Contabilística e de Relato Financeiro (NCRF) 12 – Imparidade de Ativos*(Comissão de Normalização Contabilística, 2018) (paragraph 13, point b).

To estimate free cash flows during the forecast period, two alternative approaches were employed: (a) the geometric growth rate of turnover, as suggested by Neves (2002) and Damodaran (2012); and (b) historical results, as proposed by Nguyen and Nguyen (2020), with this approach further combined with the geometric growth rate.

For the residual period, the stable growth model (Equation (41)) was adopted, a commonly used approach referenced by Damodaran (2012) and Nissim (2019). This model assumes a constant growth rate beyond the forecast period.

In determining the cost of capital, particularly the cost of equity, various cost of equity models were explored as alternatives to the widely used capital asset pricing model (CAPM).

3.2.1 Assumptions based on historical data

The financial data used in this study spans the years 2019 to 2022, with 2022 serving as the reference year for future forecasts.

To ensure the most objective and appropriate tax rate for ABC, the effective tax rate applied to the company during the reference period was determined as the basis for analysis.

Table 2. Income tax rate

	2019	2020	2021	2022
Effective tax rate	7.07%	7.27%	11.35%	14.34%
Average tax rate	10.01%			

As shown in Table 2, the effective tax rates are relatively low due to deferred taxes and autonomous taxation. Consequently, it would not be appropriate to use a rate of only 10% in the following calculations. Instead, a tax rate of 21% was applied, as stipulated by the *Código do Imposto sobre o Rendimento das Pessoas Coletivas* (*Corporate Income Tax Code*, Article 87[1]), along with a 1.5% municipal surcharge, in accordance with the terms outlined in Article 18 of the *Regime de Financiamento das Autarquias Locais e Entidades Intermunicipais* (*Financing Scheme for Local Authorities and Intermunicipal Entities*, Law No. 73/2013[2]). This tax rate will be applied consistently throughout the project wherever necessary.

Subsequently, it was essential to calculate past free cash flows to estimate their future projections. For this purpose, the financial data presented in Table 3 was used.

Table 3. Free cash flow

	2019	2020	2021	2022
(+) EBIT	210,869.35	637,716.11	396,791.39	271,553.13
(+) Amortization/depreciation	359,293.10	261,922.39	283,208.22	356,530.06
(-) CAPEX	285,571.00	64,464.55	266,781.57	1,619,047.12
(-) Δ Working capital	- 172,408.05	- 542,528.32	29,527.40	392,702.39
(=) Operating cash flow	456,999.50	1,377,702.27	383,690.64	- 1,383,666.32
(-) Tax rate x EBIT	47,445.60	143,486.12	89,278.06	61,099.45
(=) Free cash flow	**409,553.90**	**1,234,216.15**	**294,412.58**	**- 1,444,765.77**

Values in euros; income tax rate 22.5%

3.2.2 Forecasting future cash flows

To forecast future cash flows, the geometric growth rate was applied, using the company's turnover data (Table 4), as shown in Equation (6). In this equation, represents the earnings in the year 2022, represents the earnings in 2019, and *n* represents the number of years of growth, which is three years.

Table 4. Geometric growth rate

	2019	2020	2021	2022
Earnings	3,020,824.92	3,058,540.86	3,429,565.39	4,027,639.54
Geometric growth rate	**10.06%**			

Values in euros

To calculate the forecast period, we chose to compute the average of the cash flows obtained over the analysis period. This approach was selected to enhance the consistency of the results, rather than relying solely on the data from the most recent year. Additionally, as shown, the year 2022 experienced a very negative free cash flow due to significant investments in fixed assets. By adopting this strategy, we accounted for potential fluctuations over time, smoothing out temporary variations that could have distorted the forecast of future cash flows if we had only considered the last available year.

This resulted in an average historical cash flow of €123,354.21. To calculate the forecasted cash flows for the next five years, this average value was first adjusted by the geometric growth rate of 10.06%. This adjustment was repeated annually for each of the following years.

Additionally, an alternative approach was used to calculate the forecasted cash flow for 2023. A regression model based on lagged historical net results was implemented. This model, as outlined in Equation (11), was built using the financial data of 84 Portuguese companies in the metal mold manufacturing sector from the *Sistema de Análise de Balanços Ibéricos* (SABI) database, covering the period from 2017 to 2022. It is important to note that the estimation was carried out using balanced panel data, ensuring that there were the same number of periods for each observational unit.

Estimation was carried out using (a) the pooled data model, which is the ordinary least squares method (OLS); (b) the panel data model with fixed effects (FEM); and (c) the panel data model with random effects (REM). The estimations were performed using the Gretl statistical program.

After estimating the three models, it was necessary to determine which model was most appropriate to use:

(1) **OLS vs FEM**

H_0: Equal fixed effects vs H_1: There are at least two different fixed effects

Using the F-test, the two models were compared. Since p <.05, H_0 was rejected, indicating that there are different fixed effects. Therefore, the fixed effects model is preferable to the OLS model.

(2) **OLS vs REM**

H_0: Var (random effects) = 0 vs H_1: Var (random effects) > 0

where H_0 represents the non-existence of random effects and H_1 represents the existence of random effects.

Using the Breusch-Pagan test, the two models were compared. Since p >.05, H_0 could not be rejected, meaning there is not enough statistical evidence to prove the existence of random effects.

(3) **FEM vs REM**

H_0: FEM and REM are both consistent vs H_1: FEM is consistent and REM is inconsistent

The Hausman test was applied to compare the fixed effects model with the random effects model. Since p <.05, the null hypothesis (H_0) was rejected, indicating that the fixed effects model is more appropriate than the random effects model.

Therefore, after the aforementioned analysis, it was decided to use the panel data regression model with fixed effects, resulting in the following estimation model:

$$CFO_t = 458,246 + (-0.0314379) \times Earnings_{t-1} + 0.34183 \times Earnings_{t-2} + 0.138862 \times Earnings_{t-3} \quad (43)$$

where CFO_t represents the cash flow from operations in period t, $Earnings_{t-1}$ represents the net income for the period in year t-1, $Earnings_{t-2}$ represents the net income for the period in year t-2, and $Earnings_{t-3}$ represents the net income for the period in year t-3.

To estimate the free cash flows for 2023, it should be noted that the Nguyen and Nguyen (2020) model estimates cash flows from operations, not free cash flows (FCF). To address this, after implementing the estimation model in Equation (43), the following adjustments need to be made:

1. Subtract the tax rate corresponding to financial charges.
2. Subtract the cash flow resulting from the investment, as this represents the investment in fixed assets.
3. Note that the fixed assets and financial charges for 2023 should be estimated using the geometric growth rate.

Once the free cash flow for 2023 has been estimated, the geometric growth rate calculated in Table 4 will be used to calculate the estimated free cash flows for the remainder of the forecast period.

Regarding the growth rate for the residual period, the sustainable growth rate (g_u) should be used, as per the type of cash flow desired. This rate was calculated using Equations (8) and (9).

Table 5. Sustainable growth rate

	2019	2020	2021	2022
g_u	- 1.01%	- 7.75%	- 4.23%	11.01%
Average g_u	**- 0.50%**			

As shown in Table 5, this rate is negative in all years except 2022, when it is 11.01%, which is considered too high. Therefore, it was necessary to compare this rate with the economy's predicted growth rate (g_n).

According to the Bank of Portugal's projections for 2026[3], a rate of 4.14% was calculated, as shown in Equation (10). Since a company cannot be expected to grow at a higher rate than its country's economy, the economy's growth rate was selected as the sustainable growth rate for the company, as presented in Table 6.

Table 6. Economic growth rate

variable / paramater	value
GDP growth rate[1]	2.20%
Inflation rate[1]	1.90%
g_n	4.14%

(1) Obtained from the Bank of Portugal

3.2.3 Company value using the modified CAPM model

The CAPM model, modified according to Damodaran's (2014) suggestions, addresses the limitations of the traditional model, particularly its unsuitability for unlisted companies due to the absence of a market beta. Damodaran presents this modified model as a viable alternative for unlisted companies in emerging markets, such as the Portuguese market. The modification includes the addition of a risk premium to account for country risk and an adjustment to the beta to incorporate both market risk and the specific risk associated with each company.

a) Components of the CAPM model

To calculate the **risk-free rate**, we used a rate from a market with no country risk (AAA-rated) and denominated in the same currency as the valuation, the euro, which is the official currency of Portugal. The interest rate selected was the YTM of ten-year German government bonds. According to the European Central Bank[4], this rate stood at 2.18% in January 2024.

For the **market risk premium**, we used the market risk premium of a mature market (ERP_m) free of country risk. The German market was selected, as it is considered a mature market with a AAA rating. Portugal's country risk was also factored in. According to data provided by Damodaran (2023)[5], Germany had a market risk premium of 4.60%, while Portugal, with a BBB+ rating, had a country risk premium of 1.75%. This results in a total market risk premium of 6.35% (Equation (13)).

Regarding the **beta**, in accordance with Damodaran's (2014) recommendations, the total beta for the company's sector (Machinery) was obtained from the Damodaran database[6]. This beta represents an unlevered beta (β_{eu}), which was subsequently leveraged (β_e) based on the company's financial structure (Equation (21)), as detailed in Table 7. The debt beta (β_d) was then accounted for and calculated using Equation (44):

$$\beta_d = \frac{k_d - r_f}{ERP_{Portugal}} \quad (44)$$

where β_d represents the beta of the debt, k_d represents the cost of debt, r_f represents the risk-free rate, and $ERP_{Portugal}$ represents Portugal's equity risk premium.

Table 7. Leveraged company beta

variable / paramater	value
Total unlevered beta (β_{eu})[1]	2.25
Debt beta (β_d)[2]	0.299
Debt (D)[3]	1,531,890.66
Equity (E)[3]	6,112,524.26
Risk free rate (r_f)	2.18%
Tax rate (t)	22.5%
Total market risk premium (ERP_t)[4]	6.35%
Leveraged beta (β_e)	2.63
Cost of debt (k_d)	4.08%

Values in euros

(1) Collected from the Damodaran database

(2) Debt beta calculated using Equation (44)

(3) The values of equity and debt are the averages over the four-year period under analysis

(4) Represents the equity risk premium for Portugal

b) Cost of equity

Damodaran (2023) presents three approaches to calculating the cost of equity (k_e) while considering country risk. Since the company engages in international exports, the approach that incorporates this factor should be used, as shown in Equation (17). The values obtained are presented in Table 8.

Table 8. Cost of equity

variable / paramater	value
r_f	2.18%
% Company's domestic revenues	2.88%
% Average domestic revenues of companies[1]	76.39%
Lambda	0.04
$CRP_{Portugal}$[2]	1.75%
β_e	2.63
$ERP_{Alemanha}$[2]	4.60%
k_e	14.34%

(1) https://www.bportugal.pt/QS/qsweb/Dashboards

(2) https://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/ctryprem.html

c) Cost of debt

The cost of debt (k_d) was calculated by estimating a synthetic debt rating using the interest coverage ratio (Equation (34)). This rating is derived from a table provided annually by Damodaran[7]. The ratio yielded a value of 45.12, which, according to the table, corresponds to a AAA rating and a default spread of 0.59%. To calculate the cost of debt, the risk-free rate is added, and since this is an emerging market, the country default spread (CDS) of 1.31% is also included. The final value obtained for the cost of debt is 4.08% (Equation (36)).

d) Weighted average cost of capital

Finally, the weighted average cost of capital (WACC) must be determined, as it will serve as the discount rate for the forecasted free cash flows when applying the average cost method (Equation (38)). The calculation is presented in Table 9.

Table 9. Weighted average cost of capital (WACC)

variable / paramater	value
Cost of equity (k_e)	14.34%
Equity (E)1	6,112,524.26
Debt (D)1	1,531,890.66
Cost of debt (k_d)	4.08%
$E + D$	7,644,414.93
Tax rate (t)	22.5%
WACC	12.10%

Values in euros and percentages

(1) E and D correspond to the average over the four-year period under analysis

e) Company value

To calculate the value of company ABC, Equation (39) is applied, using the modified CAPM model to determine the cost of equity. The breakdown of the company value is presented in Table 10.

Table 10. Company value

	2022	2023	2024	2025	2026	2027
FCF[1]		135,767.45	149,429.84	164,467.09	181,017.56	199,233.51
FCF[2]		135,062,92	148,654,42	163,613,64	180,078,22	198,199.64
WACC		12.10%	12.10%	12.10%	12.10%	12.10%
g_n						4.14%
Residual value[3]						2,607,482.03
Residual value[4]						2,593,951.26
Company value[5]	**2,056,993.68**					
Company value[6]	**2,046,319.51**					

Values in euros

(1) Cash flow determined using the geometric growth rate

(2) Cash flow determined using the Nguyen and Nguyen (2020) forecast model

(3) Residual value obtained using the geometric growth rate

(4) Residual value obtained with cash flow determined using the Nguyen and Nguyen (2020) forecast model

(5) Company value using the FCF determined with the geometric growth rate

(6) Company value using the FCF according to the Nguyen and Nguyen (2020) model

As expected in this type of model, comparing the projected cash flows for the forecast period with the value of the residual period reveals a significant disparity, with the residual value being substantially higher. This highlights the importance of the residual value in determining the final value of the company.

By applying a discount rate of 12.10% over the five-year forecast period and a sustainable growth rate of 4.14%, the company's intrinsic value in 2022—the reference year for the valuation—amounts to €2,056,993.68 when the geometric growth rate method is used, and €2,046,319.51 when the Nguyen and Nguyen (2020) model is applied.

3.2.4 Company value through the three-component model (AECA)

To apply this model, the assumptions suggested by AECA were followed, with particular focus on the practical application of the article developed by Rojo-Ramírez (2014).

a) Components of the three-component model

The **risk-free rate** corresponds to ten-year Portuguese treasury bonds, as the company operates in Portugal. According to the European Central Bank[4], the rate in January 2024 was 2.97%.

Regarding the **market risk premium**, it was necessary to assess the return on the Portuguese index, PSI, over the period under analysis. However, the returns were low due to the impact of the COVID-19 pandemic, which significantly affected global markets, including the Portuguese market. To address this issue, the average annual return of the market between 2021 and June 2024 was used. The results, however, were also lower than expected due to various challenges, such as the war between Russia and Ukraine, rising inflation, and increasing interest rates. Despite these uncertainties, and the inability to predict future market behavior, it was decided to proceed with the data obtained.

The monthly prices of the PSI index over the period in question were obtained from Yahoo Finance[8], and the returns between the periods were calculated. An average annual return of 9.079% was obtained. By subtracting the risk-free interest rate (2.97%) from the expected market return, a market risk premium of 6.11% was derived.

The **beta** was calculated following the AECA recommendations, using a total beta to incorporate both systematic and specific risk. This beta was calculated using the historical variability of the company's return (ROE_{AT} - Return on Equity After Taxes) and the variability of the market return (R_m). The results are shown in Table 11.

The ROE is obtained from Equation (45), according to Rojo-Ramírez (2014):

$$ROE_{AT} = \frac{EBITDA - FEx - Tax}{E_m} \quad (45)$$

where ROE_{AT} represents the return on equity after taxes, *EBITDA* represents earnings before interest, taxes, depreciation, and amortization, *FEx* represents financial expenses, *Tax* represents income taxes, and E_m represents the accounting average equity for the period.

Table 11. Total beta

	2019	2020	2021	2022
ROE_{AT}[1]	9.73%	14.34%	10.11%	9.20%
σ_{ROEAT}[2]	2.36%			
	2021	**2022**	**2023**	**2024**
R_m[3]	16.34%	2.77%	11.07%	4.38%
σ_{Rm}[2]	6.27%			
β_{total}[4]	0.38			

(1) Like Rojo-Ramírez (2014), the number of years corresponding to the period under analysis was used

(2) The sample standard deviation was used because only a subset of the available years was considered, rather than the entire dataset

(3) A different period was used due to the low returns caused by COVID-19

(4) $\beta_{total} = \beta_i/\rho_{i,m} = (cov(r_i, r_m)/\sigma_m^2)/(cov(r_i, r_m)/(\sigma_i \times \sigma_m)) = \sigma_i/\sigma_m = \sigma_{ROE}/\sigma_{Rm}$ where β_i represents the sensitivity of asset i in relation to market m; $\rho_{i,m}$ represents the correlation between the profitability of asset i and the profitability of market m; $Cov(r_i,r_m)$ represents the covariance between the profitability of asset i and the profitability of market m; and asset i represents company ABC

The total beta value is considerably low, reflecting the limited volatility of the company's ROE. This suggests that the results obtained over the evaluation period are relatively stable, with consistent returns.

b) Cost of equity

The cost of equity was calculated using Equation (27) and is presented in Table 12 for analysis.

Table 12. Cost of equity

variable / paramater	value
r_f	2.97%
P_M	6.11%
β_{total}	0.38
k_e	11.38%

c) Cost of debt

To calculate the cost of debt, we followed Damodaran's (2014) recommendations, as described in the previous model, determining the default spread using a synthetic rating.

For the interest coverage ratio, we used the average of both variables over the four-year period in question, resulting in a value of 45.12. This corresponds to a default spread of 0.59%, according to the Damodaran database[7]. By adding the risk-free rate (2.97%) and the country default spread (1.31%), the final cost of debt is 4.87%, as calculated in Equation (36).

d) Weighted average cost of capital (WACC)

The WACC was calculated using Equation (38) and is presented in Table 13.

Table 13. Weighted average cost of capital

variable / paramater	value
k_e	11.38%
E[1]	6,112,524.26
D[1]	1,531,890.66
k_d	4.87%
$E+D$	7,644,414.93
t	22.5%
$WACC$	9.86%

Figures in euros
(1) E and D correspond to the average for the four-year period under analysis

e) Company value

To calculate the value of company ABC, Equation (39) was applied, and the company's value was determined using the three-component model to calculate the cost of equity. The calculations are presented in Table 14.

Table 14. Company value

	2022	2023	2024	2025	2026	2027
FCF[1]		135,767.45	149,429.84	164,467.09	181,017.56	199,233.51
FCF[2]		135,062.92	148,654.42	163,613.64	180,078.22	198,199.64
WACC		9.86%	9.86%	9.86%	9.86%	9.86%
g_n						4.14%
Residual value[3]						2,105,055.22
Residual value[4]						2,094,131.65
Company value[5]	**1,935,889.96**					
Company value[6]	**1,925,844.22**					

Values in euros; The residual value is calculated on the assumption that there is no sustainable growth rate

(1) Cash flow determined using the geometric growth rate

(2) Cash flow determined using the Nguyen and Nguyen (2020) forecast model

(3) Residual value obtained using the geometric growth rate

(4) Residual value obtained with cash flow determined using the Nguyen and Nguyen (2020) forecast model

(5) Company value using the FCF determined with the geometric growth rate

(6) Company value using the FCF according to the Nguyen and Nguyen (2020) model

Applying a discount rate of 9.86%, the company's value in 2022 is €1,935,889.96 when the free cash flows are obtained using the geometric growth rate. When the cash flows are derived using the Nguyen and Nguyen (2020) model, the company's value is €1,925,844.22.

3.2.5 Company value through the build-up model

This model follows the recommendations of Professor Roger Ibbotson, as outlined by Ballwieser and Wieser (2010). The data used in this model is sourced from the Duff & Phelps (Kroll) databases, a well-established and respected US consulting firm.

a) Components of the build-up model

In June 2024, Duff & Phelps[9] recommended a **risk-free rate** of 2.50%. This rate, based on German treasury bonds with a maturity of 15 years, is the rate recommended for use by European companies.

The **market risk premium** is also determined by Duff & Phelps[9], and for European companies in June 2024, it is set at 5.5%.

The **risk premium based on company size** was obtained from the 2022 annual report provided by Duff & Phelps, following the methodology of Feldman and Feldman (2023). According to these authors, the premium is typically calculated based on the company's market capitalization. However, since the company being valued is private and lacks this information, the book value of equity was used instead, following the recommendation of Pratt and Grabowski (2014). This approach yielded a size premium of 11.17%, a relatively high value due to the company's classification as a small business in the bottom category (10z) of smaller companies. This category corresponds to higher risk and, consequently, a higher cost of capital. As Pratt and Grabowski (2014) point out, smaller companies generally face greater risk, justifying the higher premium.

The **industry risk premium** was calculated using a beta based on data from companies within the same industry. Damodaran's database[10] was used to determine the unleveraged beta of the metal mold manufacturing industry (Machinery), which was 0.91. This beta was then adjusted to reflect the specific sector in Portugal (metal mold manufacturing) using the bottom-up beta method, resulting in a leveraged beta of 2.24. To calculate the industry risk premium, Equation (33) was applied, yielding an industry risk premium of 6.79%. It is important to note that the debt beta was not used, as its inclusion would have resulted in a negative beta. Following Beneda's (2003) approach, the debt beta was excluded from this calculation.

Finally, the **specific risk premium associated with the company** was calculated. This is a highly subjective risk, as it largely depends on the appraiser's experience and knowledge of the company. According to Boudreaux et al. (2012) and Pratt and Grabowski (2014), this specific risk is influenced by various factors, such as: (a) the risk of the industry in which the company operates, (b) the volatility of the company's earnings, (c) the company's level of debt, (d) dependence on key customers, (e) diversification of the company's activities, and other company-specific factors. Since no studies have been conducted to analyze this relationship between the company and the sector, it was decided to assign a 0% premium, consistent with the approach of Tang (2023).

b) Cost of equity

The cost of equity (Table 15) was calculated using Equation (32).

Table 15. Cost of equity

variable / paramater	value
r_f	2.50%
ERP	5.50%
PR_{size}	11.17%
$PR_{industry}$	6.79%
$PR_{company}$	0%
k_e	25.96%

c) Cost of debt

The cost of debt was calculated again according to Damodaran's (2014) recommendations[7], as described in the previous model. The same default spread of 0.59% was applied. By adding the risk-free interest rate (2.50%) and the country default spread (1.31%), a value of 4.40% was obtained, as per Equation (36).

d) Weighted average cost of capital (WACC)

The WACC (Table 16) was calculated using Equation (38).

Table 16. Weighted average cost of capital

variable / paramater	value
k_e	25.96%
$E1$	6,112,524.26
$D1$	1,531,890.66
k_d	4.40%
$E+D$	7,644,414.93
t	22.5%
WACC	21.44%

Values in euros

(1) E and D correspond to the average for the four-year period under analysis

e) Company value

Once again, Equation (39) was applied to obtain the value of company ABC, this time using the build-up model to calculate the cost of equity. The results are shown in Table 17.

Table 17. Company value

	2022	2023	2024	2025	2026	2027
FCF[1]		135,767.45	149,429.84	164,467.09	181,017.56	199,233.51
FCF[2]		135,062.92	148,654.42	163,613.64	180,078.22	198,199.64
WACC		21.44%	21.44%	21.4%	21.44%	21.44%
g_n						4.14%
Residual value[3]						1,199,165.96
Residual value[4]						1,192,943.24
Company value[5]	**917,504.43**					
Company value[6]	**912,743.30**					

Values in euros

(1) Cash flow determined using the geometric growth rate

(2) Cash flow determined using the Nguyen and Nguyen (2020) forecast model

(3) Residual value obtained using the geometric growth rate

(4) Residual value obtained with cash flow determined using the Nguyen and Nguyen (2020) forecast model

(5) Company value using the FCF determined with the geometric growth rate

(6) Company value using the FCF according to the Nguyen and Nguyen (2020) model

As seen in the two previous valuations, the difference between the expected cash flows and the residual value is once again evident.

Applying a constant discount rate of 21.44% and an economic growth rate of 4.14%, the company has an intrinsic value of €917,504.43 in 2022, using the FCF forecasting method with the geometric growth rate. With the FCF forecasting method based on the Nguyen and Nguyen (2020) model, the company's value is €912,743.30.

3.2.6 Summary of the various approaches applied

After evaluating company ABC for the year 2022, it is clear that the valuation values obtained do not vary significantly. Among the models analyzed, the three-component model and the modified CAPM model yield closer values, while the build-up model shows lower values. Regardless of the model used, the final valuation is not considerably high, mainly due to the relatively low expected cash flows. These cash flows were calculated based on the average of the company's past cash flows from 2019 to 2022, which are also relatively low.

Across the three models, the importance of defining an appropriate discount rate for the application of the DCF method is evident. A correct and adequate discount rate ensures that the company's valuation is as accurate and objective as possible. The higher the discount rate, the greater the cost to the company, and consequently, the lower its value.

It was also found that the value of the residual period plays a crucial role in the total value of the company, regardless of the model used.

Finally, in terms of estimating future cash flows, it can be concluded that both models produced very similar results. This reinforces their credibility and reliability, leading to an equally similar overall company value.

4. Discussion of results and conclusions

The aim of this work was to outline the evaluation process of a small- and medium-sized enterprise (SME), using a Portuguese SME, ABC, as a case study. The Discounted Cash Flow (DCF) methodology was employed to determine the company's value, with particular emphasis on choosing the correct discount rate for the cash flows in order to obtain the most realistic estimate of the company's value. Since the goal was to value the company as a whole, the free cash flows were determined using the weighted average cost of capital (WACC) as the discount rate, which necessitated the calculation of both the cost of equity and the cost of debt.

To determine the cost of equity, three alternative models to the traditional capital asset pricing model (CAPM) were applied: the modified CAPM (incorporating Damodaran's suggestions), the AECA model, and the build-up model proposed by Roger Ibbotson.

Like any financial model, these models have both advantages and disadvantages in their application. The primary advantage of all three models is that they can be applied to private companies, particularly small and medium-sized enterprises (SMEs), allowing for a cost of capital that better reflects the business realities of these companies. This contrasts with the use of models designed for publicly listed companies, such as the traditional CAPM, which may not be as suitable for private entities. However, a significant limitation of these models is the difficulty in obtaining reliable financial data from private companies, as they are not required to publicly disclose their financial information.

The specific advantages and disadvantages of each model will be discussed individually below.

In the CAPM model, according to Damodaran's modifications, a sector beta is used to calculate the cost of equity. By adjusting this beta to the company's financial structure through the bottom-up approach, more accurate values are obtained that better reflect the financial reality of the company under analysis. This approach is preferred over relying solely on sector data provided by Damodaran, which is based on the American market. Additionally, this model incorporates the country risk factor by considering a default risk rating, acknowledging that the greater a country's financial stability and credibility, the lower its risk. As a result, emerging markets face higher country risk than mature markets, which tend to have a considerably lower risk of default.

However, a disadvantage of the CAPM model with Damodaran's modifications is that it typically results in lower cost of equity values, and consequently, a higher company valuation. This can be problematic, as it may lead to an overvaluation of the company. In turn, this could cause directors to overestimate the company's worth, which can be detrimental in scenarios involving mergers, acquisitions, or sales. Another issue is that the reference market for the values recommended by Damodaran is the United States, which may not reflect the specific conditions and characteristics of other markets, such as the Portuguese market in this case.

Regarding the three-component model, Rojo-Ramírez et al. (2012) argue that the cost of equity rate derived from this model should be used when valuing companies using the discounted cash flow method. This model accounts for both non-diversifiable risk and the effect of illiquidity. It allows for the consideration of both market risk and specific risk. Since SMEs are privately held, they cannot reduce or eliminate the specific risk through diversification as public companies can. Thus, the three-component model is especially suitable for private companies because it incorporates factors such as the lack of liquidity and the specific risks inherent to each business—elements that other models often overlook.

By factoring in these elements, the cost of equity obtained through this model is more representative of the realities faced by private companies. This typically results in a higher cost of equity and, consequently, a lower valuation compared to models like the CAPM, as noted by Rojo-Ramírez et al. (2012). This approach offers a valuation that is closer to the company's true value, providing managers with a more accurate basis for making future strategic and administrative decisions.

To calculate the market risk premium and the specific risk premium, data from the relevant market is used. However, this data may not always be from mature markets, but rather from emerging markets such as Portugal, which carry a higher level of risk. Consequently, if the market is negatively affected by global events—such as the COVID-19 pandemic or geopolitical conflicts—the results may be skewed, potentially leading to inaccurate assessments. This presents a disadvantage in applying the model, particularly as its knowledge and use are still somewhat limited within the scientific community.

The build-up model allows for the inclusion of several relevant risk premiums in the calculation of the cost of equity, such as market risk, industry risk, company size risk, and company-specific risk. By breaking down the cost of equity into these different components, the evaluator can better understand how each variable contributes to the company's total cost. Additionally, the model benefits from using a full information beta, which is derived from data on companies in the same industry to assess both the risk and specific characteristics of that industry.

One of the main disadvantages of the build-up model is the high degree of subjectivity involved in determining the various risk premiums, such as the industry risk premium, the risk premium associated with the company's size, and the specific risk premium. The latter, in particular, presents significant challenges. The choices made by assessors can have a considerable impact on the results, potentially leading to distortions (Dzuričková et al., 2015). Moreover, justifying the values assigned to each premium can be complex. As a result, the way these premiums are calculated and applied may lead to an underestimation or overestimation of the cost of equity, which could negatively affect the company's future decisions. Additionally, the build-up method is limited by the lack of empirical evidence validating its credibility and reliability in practical applications.

The models analyzed in this study incorporate different variables for calculating the cost of equity, drawing on varying data sources—some from the American market (as suggested by Damodaran) and others specific to the Portuguese market. Additionally, the number of risk premiums considered differs across models, with the build-up model accounting for the most risk factors. These distinctions result in variations in the calculated cost of equity, which directly influence the company's valuation. Specifically, a higher cost of equity leads to a higher discount rate being applied to projected cash flows, thereby lowering the company's estimated present value.

According to Rojo-Ramírez et al. (2012) and Rojo-Ramírez (2014), the CAPM model is not ideal for private companies, as it often results in a lower cost of equity, leading to an overvaluation of the company. This is evident in the results, where the value calculated using the CAPM model exceeds those obtained with the other two models, supporting the authors' assertion.

It is also evident that the cost of equity derived from the CAPM model is lower than that calculated using the build-up model, consistent with the findings of Michalak (2014), Rafanelli (2016a, 2016b), and Gnap (2023). However, contrary to the expectations of Rojo-Ramírez et al. (2012) and Rojo-Ramírez (2014), the cost of equity obtained using the three-component model was not higher than that from the CAPM. This discrepancy can be attributed to various adverse global events that impacted financial markets. Specifically, a low total beta was recorded due to the low volatility of the company's returns, indicating minimal fluctuations during the review period. Moreover, as the authors did not account for perpetuity growth in the residual period, their analysis reached the same conclusion: the company's value under the CAPM model exceeds that of the three-component model.

Lastly, the value of the company calculated using the build-up method is the lowest among the models, as it accounts for a greater number of risk factors and their associated risk premiums, thereby increasing the cost of capital and consequently reducing the company's valuation.

Regardless of the equity model applied, the company's intrinsic value remains relatively low. This indicates that the company is not generating sufficient value from its operations, as reflected in the cash flow figures over the past few years. It may therefore be necessary to revise its operating policies to achieve higher returns in the near future.

In conclusion, as noted by Nissim (2019), the terminal value has a significant impact on the overall company valuation, often representing substantially higher values compared to the projected cash flows.

As mentioned above, the main objective of this work was to outline the evaluation process for an SME, with a particular focus on the methodology for calculating the cost of equity. While the CAPM model is widely used in the literature due to its simplicity and applicability, it is not fully suited to the context of private SMEs like the company analyzed, as it was primarily developed for public companies.

This study, therefore, aims to contribute to the limited literature on SMEs and private companies by proposing alternative models better aligned with their realities. By applying any of the three models discussed, both market risk and the specific risks faced by the company are considered, offering managers a more realistic view of the company's cost of capital. An accurate estimate of this cost is crucial for investors and shareholders, as it represents the minimum return expected on their investments.

In the context of company valuation, the cost of capital is fundamental, as it serves as the discount rate in the discounted cash flow model. A precise and well-supported valuation enables managers to make more informed strategic and investment decisions, offering a clearer understanding of the relationship between risk and expected return. By gaining a better grasp of the company's intrinsic value, managers can more easily acquire third-party capital, benefiting from a more accurate perception of the company's real worth and financial stability. Furthermore, the valuation process fosters trust and transparency among all stakeholders, strengthening the company's credibility and demonstrating management's commitment to maintaining an open and responsible relationship.

A limitation of this study is the scarcity of available information, as the company is not listed on the market and is not required to disclose financial data. The accounting information used is from 2022, which is the most recent data available. Additionally, there were challenges in determining certain variables necessary for implementing the proposed models. Another important limitation is the high level of subjectivity involved in applying the build-up model, as the assumptions and decisions made by the evaluator can introduce biases, potentially leading to less reliable estimates. Furthermore, the historical period used (2019-2022) is marked by significant global events, such as the COVID-19 pandemic, multiple wars, rising interest rates, and inflation, which are expected to negatively affect the results.

Finally, it is worth noting the lack of empirical validation for the three-component and build-up models, which do not have the same theoretical and practical consolidation as the CAPM model. This limitation in empirical validation may affect the reliability of the results and raise concerns about the suitability of these models for SMEs.

In terms of future suggestions, one potential improvement would be to conduct a risk analysis of the variables studied, particularly by employing Monte Carlo simulation. This technique would provide insights into how changes in input variables might affect the company's value. In other words, it would help identify the variables that contribute most significantly to value creation. To implement this approach, it would be necessary to first determine the key variables for analysis and assign an appropriate probability distribution to each. Conducting thousands of iterations would enhance the credibility of the analysis, allowing for a more robust evaluation of the results.

Another suggestion for improvement would be to assess the company using additional valuation methods alongside the discounted cash flow model. Specifically, the economic value added (EVA) model could offer a complementary and more comprehensive perspective on the company's value. Furthermore, the EVA model's reduced reliance on residual value can contribute to a more stable and reliable valuation.

Footnotes

[1] Source: https://diariodarepublica.pt/dr/legislacao-consolidada/lei/2014-64205634-836144690
[2] Source: https://diariodarepublica.pt/dr/legislacao-consolidada/lei/2013-105795409-131574593
[3] Source: https://www.bportugal.pt/page/projecoes-economicas
[4] Source: https://www.ecb.europa.eu/stats/financial_markets_and_interest_rates/long_term_interest_rates
[5] Source: https://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/ctryprem.html
[6] Source: https://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/totalbeta.html
[7] Source: https://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/ratings.html
[8] Source: https://finance.yahoo.com/quote/PSI20.LS/history/
[9] Source: https://www.kroll.com/en/insights/publications/cost-of-capital/recommended-eurozone-equity-risk-premium-corresponding-risk-free-rates
[10] Source: https://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/Betas.html

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